DETERMINACY AND INDETERMINACY

Equations of Static Equilibrium

(*a*) Plane Structure (2D-structure): A structure is said to be 2-D structure or plane structure when all the members or forces in the structure are in one plane only.

Some examples of plane structures are:

(*i*) Beams (*ii*) Plane trusses
(*iii*) Plane frame (*iv*) Cables
(*v*) Arches, etc.

The equation of equilibrium for a planar structure are

$$\Sigma F_x = 0 \qquad \Sigma F_y = 0 \qquad \Sigma M = 0$$

(*b*) Space Structure (3D-structure): A structure is said to be 3D structures in which members and forces are in 3D.

Some examples are,

(*i*) Space truss (*ii*) Space frame etc.

The equations of equilibrium are:

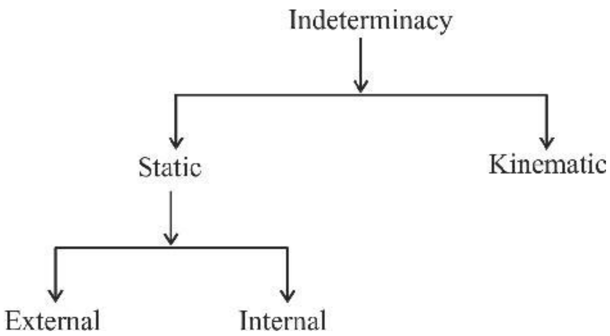
$$\Sigma F_x = 0 \qquad \Sigma M_x = 0$$
$$\Sigma F_y = 0 \qquad \Sigma M_y = 0$$
$$\Sigma F_z = 0 \qquad \Sigma M_z = 0$$

Statically Determinate Structures:

- A structure is said to be statically determinate structure if the condition of equilibrium are sufficient to fully-analyze the structure.
- B.M. and S.F. at a section are independent of the material properties and cross-sectional dimensions of the components of the structure.
- No stresses are induced due to temperature changes and lack of fit.
  e.g.-Simply supported beam, cantilever beam etc.

Statically Indeterminate Structures OR Redundant Structures:

- A structure is said to be statically indeterminate structure if the conditions of equilibrium aren’t sufficient to fully analyze the structure.
- B.M. and S.F. at a section depends on the material properties and cross sectional dimensions of the components of the structure.
- Stresses are induced due to temperature changes and lack of fit.
  e.g. - Fixed beam, continuous beam etc.



(A) STATIC INDETERMINACY

- If there are '*n*' unknown like moment, shear, axial force etc. even after applying the laws of static equilibrium, the structure is said to be redundant to '*n*' degree.

(i) **External Redundancy:** If the external support reactions cannot be determined by using the equations of static equilibrium, the structure is termed as externally redundant.

- It is equal to number of external reaction components in addition to number of equilibrium conditions.

(ii) **Internal redundancy:** If the internal member forces provided to safely resist the external forces cannot be determined by using the equation of static equilibrium, the structure is teamed as internally redundant.

- It refers to geometric stability of the structure.

Total Indeterminacy: $D_S = D_{Si} + D_{Se} \rightarrow$ Degree of external static indeterminacy

(D_{Si} ↓ Degree of internal static indeterminacy)

or D_S = No. of unknown forces – Equations of static equilibrium available

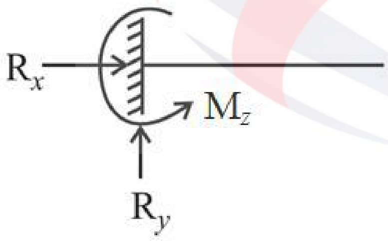
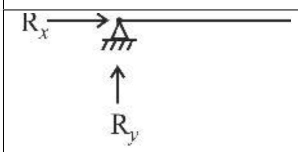
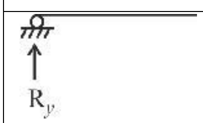
D_{Se} = Total no. of support reactions – No. of equations of static equilibrium available

$= R - 3 \rightarrow$ For 2D structure

$= R - 6 \rightarrow$ For 3D structure

Support Reactions:

(*a*) Plane Structure :

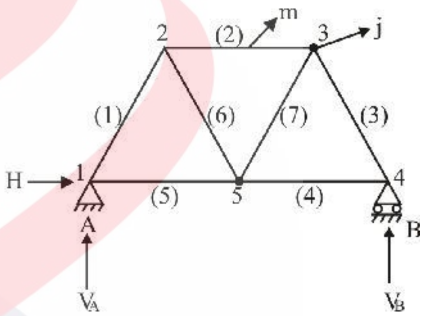
	Support reactions are R_x, R_y and M_z (3 no.)
	Support reactions are R_x and R_y (2 no.)
	Support reaction is R_y (1 no.)

(*b*) Space Structure:

	Support reactions are $R_x, R_y, R_z, M_x, M_y, M_z$ (6 no.)
	Support reactions are R_x, R_y, R_z, (3 no.)
	Support reaction is R_y (1 no.)

Methods to determine static indeterminacy of a structure.

Case I: Pin jointed plane trusses:



Trusses are pin-joined frames which carry only axial forces.

Static indeterminacy (SI):

SI= (m + r)-2j

Where, m = Number of members

r = Number of reactions

j = Number of joints

- Total No. of unknowns = $m + r$

  Number of equations available at joint = 2j

  If SI = 0, Structure is statically determinate

  SI> 0, Structure is statically indeterminate

  SI< 0, Structure is kinematically unstable

For the figure given above, the static indeterminacy (SI) is calculated as

$$\text{S.I.} = m + r_e - 2j$$

$m = 7$ $\quad r_e = 3 \quad j = 5$

$\therefore$ S.I. = 7 + 3 – 2 × 5 = 0

Externally redundant or indeterminate: for plane truss.

For a planar structure, there are three equations of equilibrium

$\boxed{\sum H = 0}$, $\boxed{\sum V = 0}$ & $\boxed{\sum M = 0}$

Hence, if Number of reactions = r then,

If r> 3, Structure is externally indeterminate

r = 3, Structure is externally determinate

r< 3, Structure is externally Kinematically unstable

D_{Se} (Degree of external indeterminacy) = r – 3

Internally indeterminate:

If $m > (2j - 3)$, Structure is internally indeterminate

$m = (2j - 3)$, Structure is internally determinate

$m < (2j - 3)$, Structure is internally kinematically unstable.

D_{Si} (Degree of internal indeterminacy) = $m - (2j - 3)$

So, D_S (Total degree of static indeterminacy) = $D_{Se} + D_{Si}$

$= r - 3 + m - (2j - 3) = m + r - 2j$

If a rigid frame has hybrid joints such as presence of internal hinge, link, roller etc. than some of the internal reactions will be released hence D_{si} (Degree of internal indeterminacy) will be released. If r_r is total number of released reactions then value of r_r is calculated as

$r_r = m' - 1$ { for 2D structure}

$r_r = 3(m' - 1)$ {for 3D structure}

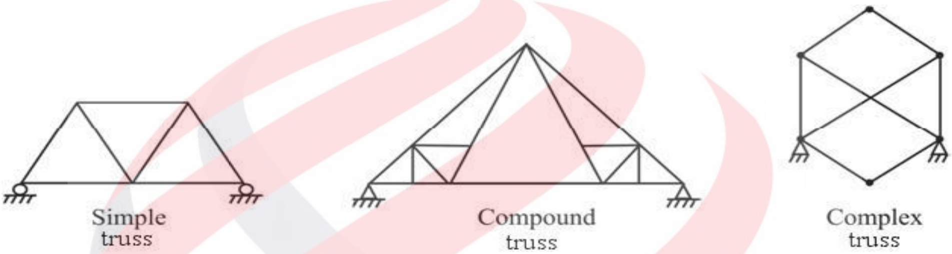
m' = number of members meeting at internal hinge.

Types of Truss

(*i*) Simple truss: When two bar and one joint are progressively added to form a truss, the truss is called simple truss.

(*ii*) Compound truss: These are the trusses formed by connecting two simple truss by a set of joints and bars.

(*iii*) Complex truss: There is no joint in the truss where only two bars meet.

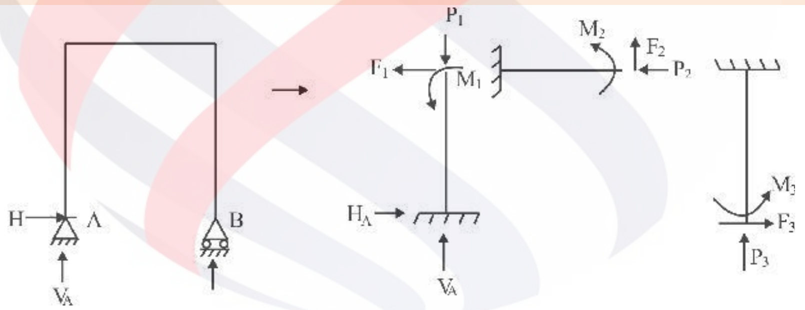


→ if hybrid joints are present then,

$D = m + r - 2j - r'$

r' = number of reactions released

Case II: Rigid jointed plane frame:



- Unlike a pin jointed frame, in rigid jointed frame, a truss member resist three stress resultant (Axial, shear force and bending moment)
  Hence, [Total number of internal stresses = 3*m*]
- Total number of unknowns = 3*m* + r
- Also at every joint 3 equations of equilibrium are available

  $\boxed{\sum H = 0}$, $\boxed{\sum V = 0}$, $\boxed{\sum M = 0}$

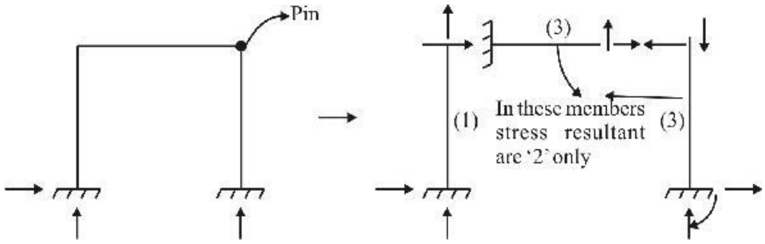
  $\boxed{\text{Total no. of equations available} = 3j}$
- Statically indeterminacy (*SI*)

  $\boxed{SI = (3m + r) - 3j}$

  Note: This equation cannot be used as a generalized formula for all types of frame. In such cases

  $\boxed{SI = \text{Total number of unknown} - \text{Total number of equations available}}$

e.g.



Hence, If $SI>0$; Structure is statically indeterminate

$SI = 0$; Structure is statically determinate

$SI<0$; Structure is statically kinematically unstable

Externally indeterminate:

If, $r>3$;Structure is externally indeterminate

$r = 0$; Structure is externally determinate

$r< 3$; Structure is externally Kinematically unstable.

$D_{se} = r - 3$

Internally indeterminate:

If, $3m> (3j - 3)$; Structure is internally indeterminate

$3m = (3j - 3)$; Structure is internally determinate

$3m< (3j - 3)$; Structure is internally kinematically unstable

$D_{si} = 3m - (3j - 3)$

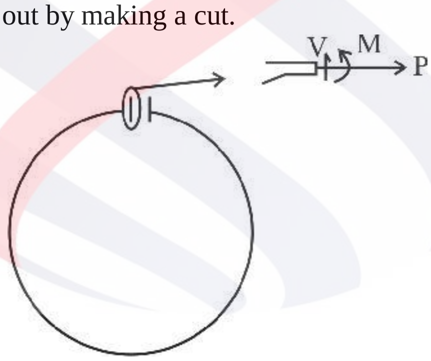
If hybrid joints are present then, $D_s = 3m + r - 3j - r'$

Where, r' = Released reactions

Ring Concept

Let us take a general plane frame member. It may be assumed as a ring subjected to loads and because of loads, it deforms. Therefore, internal forces are developed in the ring.

Now, these internal forces can be found out by making a cut.



- A cut releases three internal forces, shear (V), axial force (P) and bending moment (M) at a section

Hence, total unknown member forces = 3

→Applying above concept for closed frames, static indeterminacy can be calculated as,

D_{Si} (Internal Indeterminacy) = $3C - r_r$ (For 2D structure)

D_{Si} (Internal Indeterminacy) = $6C - r_r$ (For 3D structure)

D_{Se} (External Indeterminacy) = R – 3 (For 2D frames)

D_{Se} (External Indeterminacy) = R – 6 (For 3D frames)

Where, C = Number of loops (rings)

$r_r = \Sigma(M'_j - 1)$ (For 2D)

$= 3\Sigma(M'_j - 1)$ (For 3D)

M'_j = Number of member connecting with j' number of joints

j' = Number of hybrid joint.

Case III: Pin Jointed Space Truss :

- In a 3-dimensional pin-jointed truss, all the members carry axial force only and hence the number of total unknown internal forces = *m* and let total of reactions be '*r*'.
  Hence, total number of unknowns = $m + r$
- Number of equations available at joint
  $= 3j\left[\sum Fx = 0, \sum Fy = 0, \sum Fz = 0, \text{ Moment equations are automatically satisfied}\right]$
- Statically indeterminacy : $\boxed{SI = m + r - 3j}$
- Hence, [$SI > 0$, $SI = 0$, $SI < 0$ represents statically indeterminate, determinate and Kinematically unstable structure respectively]

If hybrid joints are present then

$D_S = m + r - 3j - r'$

r' = number of reactions released

External Indeterminacy:

Number of equation of equilibrium is six as given by:

$\boxed{\sum F_x = 0}$ $\boxed{\sum M_x = 0}$

$\boxed{\sum F_y = 0}$ $\boxed{\sum M_y = 0}$

$\boxed{\sum F_z = 0}$ $\boxed{\sum M_z = 0}$

Hence,

If, $r > 6$; Structure is externally indeterminate

$r = 6$; Structure is externally determinate

$r < 6$; Structure is externally Kinematically unstable

Internal Indeterminacy:

If, $m > (3j - 6)$; Structure is internally indeterminate

$m = (3j - 6)$; Structure is internally determinate

$m < (3j - 6)$; Structure is internally kinematically unstable