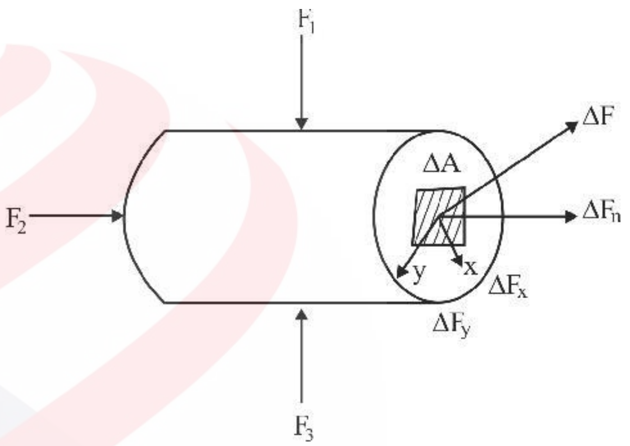
SIMPLE STRESSES AND STRAINS

STRESS (±):

It is the internal resistance offered by a body against the deformationnumerically, it is given as force per unit area.

Stress on elementary area ΔA,

i.e. $\boxed{\sigma = \lim_{\Delta A \to 0} \frac{\Delta F}{\Delta A} = \frac{dF}{dA}} (N/m^2)$ This unit is called



Pa(Pascal)

In case of normal stress *d*F always ⊥ (perpendicular) to area *d*A.

Pascal is a small unit in practice. These units are generally used

$$1kPa = 10^3 Pa = 10^3 N/m^2$$

$$1MPa = 10^6 Pa = 10^6 N/m^2$$

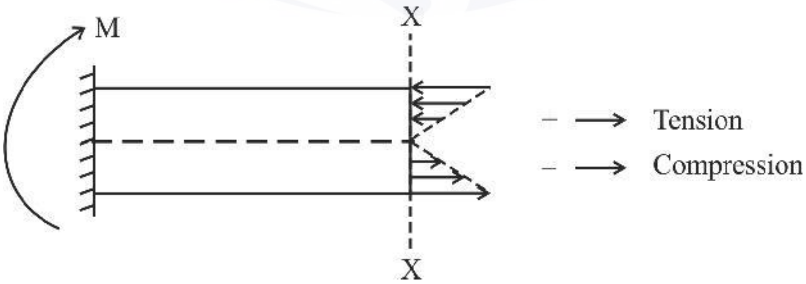
$$1GPa = 10^9 Pa = 10^9 N/m^2$$

1. **Normal Stress:** It may be tensile or compressive depending upon the force acting on the material.

   Tensile and compressive stresses are called **direct stresses.**

   When, $\sigma > 0$, Tensile
   When, $\sigma < 0$, Compressive

- The other types of normal stress is bending normal stress.



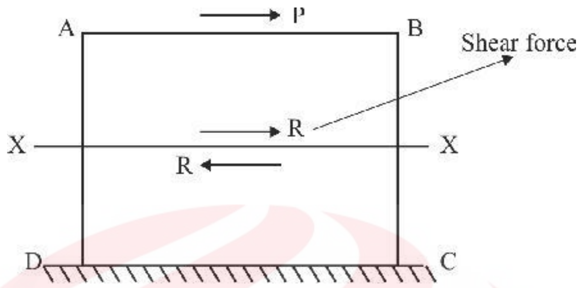
Bending stress are linearly distributed from zero at neutral axis to maximum at surface.

- In bending, the cross-sectional area rotates about transverse axis and the axis about which the cross-sectional area rotates is called neutral axis hence in bending, neutral axis is always transverse axis.

2. **Shear Stress (^{2}):** It is the intensity of shear resistance along a surface (Let X-X).

$$\boxed{\tau = \frac{Shear\ \ force}{Shear\ \ Area}}(N/m^2)$$

In case of shear stress force always parallel to the sheared area *i.e.* P is parallel to sheared area in figure.



3. **Conventional or Engineering Stress (±$_0$):** It is defined as the ratio of load (P) to the original area of cross-section (A_0):

$\therefore$ $$\boxed{\sigma_0 = \frac{P}{A_0}}$$

4. **True Stress (±):** It is defined as the ratio of load (P) to the instantaneous area of cross-section (A):

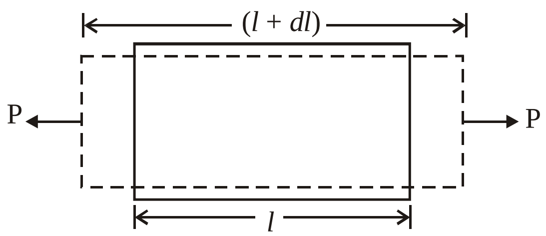
$\therefore$ $\boxed{\sigma = \frac{P}{A}}$ $or,$ $\boxed{\sigma = \sigma_0(1+\varepsilon)}$ Whereε = strain $[Al = A_0 l_0]$ Initial volume = Final volume

$l = l_0(1+\varepsilon)$

STRAINS (£):

It is defined as the change in length per unit length. It is a dimensionless quantity.

i.e. $$\boxed{\varepsilon = \frac{\text{change in length}}{\text{original length}} = \frac{dl}{l}}$$



1. **Conventional or Engineering strain:** It is defined as the change in length per unit original length.

$$\boxed{\varepsilon = \frac{l - l_0}{l_0}}$$

Where,

l = Deformed length l_0 = Original length

e.g. from above figure.

$$\varepsilon = \frac{l + dl - l}{l} \qquad \boxed{\varepsilon = \frac{dl}{l}}$$

2. Natural Strain: It is defined as the change in length per unit instantaneous length.

$$\boxed{\bar{\varepsilon} = \int_{l_0}^{l} \frac{dl}{l} = ln\frac{l}{l_0} = ln(1+\varepsilon) == \ln\left(\frac{A_0}{A}\right) = 2\ln\left(\frac{d_0}{d}\right)}$$

Also, $\because$ $\bar{\varepsilon} = \ln(1+\varepsilon)$

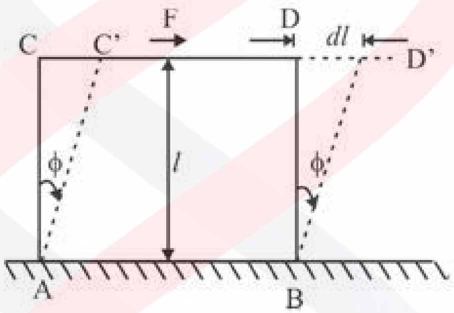
$\Rightarrow 1+\varepsilon = e^{\bar{\varepsilon}}$

$\Rightarrow \varepsilon = e^{\bar{\varepsilon}} - 1$

Volume of the specimen is a assumed to be constant during plastic deformation

$\because$ $\boxed{A_0L_0 = AL}$ -Valid till neck formation.

3. Shear Strain (¤): It is the strain produced under the action of shear stresses.



Shear Strain = tan ϕ

For small strain, $\boxed{\tan\phi \approx \phi}$

From figure, $\Delta ACC'$ or $\Delta BDD'$

$$\tan\phi = \frac{dl}{l} = \frac{CC'}{l}$$

$$\boxed{\phi = \frac{dl}{l} = \frac{\text{Transverse displacement}}{\text{Distance from lower face}}}$$

➢ Shear strain cause deformation in shape but volume remains same.

4. Superficial strain (£$_s$): It is defined as the change in area of cross section per unit original area.

$$\boxed{\varepsilon_s = \frac{A - A_0}{A_0}}$$

Where, A = Final area A_0 = Original area

5. **Volumetric Strain (ε_v):** It is defined as the change in volume per unit original volume.

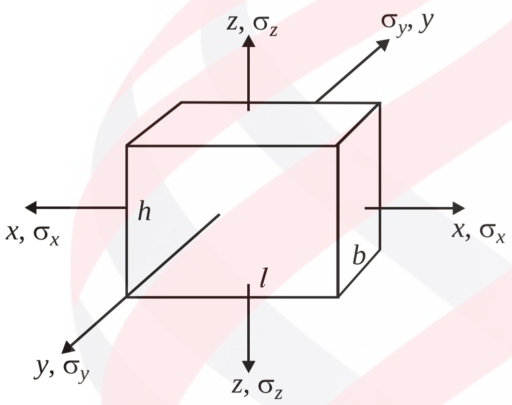
$$\varepsilon_V = \frac{V - V_0}{V_0}$$

Where, V = Final volume V_0 = Original volume

- Stress andstrain are tensor (*neither vector nor scalar*) of 2nd order.

$$\text{Volumetric strain } \varepsilon_V = \varepsilon_x + \varepsilon_y + \varepsilon_z$$

Volumetric strain for various shapes:



(*i*) Rectangular body:

V = lbh on partial differentiation

$$\delta V = \delta l(b.h) + \delta b(l.h) + \delta h(b.l)$$

$$\varepsilon_V = \frac{\delta V}{V} = \frac{\delta l}{l} + \frac{\delta b}{b} + \frac{\delta h}{h}$$

$$\varepsilon_V = \varepsilon_x + \varepsilon_y + \varepsilon_z$$

Note: $\varepsilon_x, \varepsilon_y, \varepsilon_z$ are the strain corresponding to the stresses $\sigma_x, \sigma_y, \sigma_z$ in x-direction, y-direction, z-direction respectively

$$\varepsilon_V = \frac{\sigma_x + \sigma_y + \sigma_z}{E}(1 - 2\upsilon)$$

$\upsilon \rightarrow$ POISSON Ratio

υ= 0.5 For rubber

(*ii*) For cylindrical body:

$$V = \frac{\pi}{4} d^2 l$$

$$\delta V = 2\, dl\,.\,\delta d.\frac{\pi}{4} + \frac{\pi}{4} d^2 \delta l$$

$$\varepsilon_V = \frac{\delta V}{V} = 2\frac{\delta d}{d} + \frac{\delta l}{l}$$

$$\boxed{\varepsilon_V = 2\varepsilon_d + \varepsilon_l}$$

(*iii*) For spherical body

$$\boxed{\varepsilon_V = 3\frac{\delta d}{d}} \qquad \boxed{V = \frac{4}{3}\pi r^3} \qquad d = 2r$$

Gauge Length: It is that portion of the test specimen over which extension or deformation is measured. i.e. this length is used in calculating strain value.

Poisson's ratio $\left(\upsilon \text{ or } \frac{1}{m}\right)$: Value of v varies between (–1 to 0.5)

The ratio of the lateral strain to longitudinal strain is called the Poisson's ratio.

$$\boxed{\upsilon = \frac{\text{Lateral strain}}{\text{Longitudinal strain}}} \quad \text{or} \quad \boxed{\upsilon = \frac{-\left(\frac{\delta d}{d}\right)}{\left(\frac{\delta l}{l}\right)}}$$

- For a given material, the value of 'υ' is constant throughout the linearly elastic range.
- For most of the metals the value of 'υ' lie between 0.25 – 0.42
- 'υ' varies from (–1 to 0.5)

Note:'υ' for ductile material is greater than 'υ' for brittle metals.

Table

Material	Value of ‘3’	Remarks
Cork	0	∴ Used in bottle to withstand pressure
Foam	–1	
Rubber	0.5	
Concrete	0.1 – 0.2	
C.I.	0.23 – 0.27	

For cork $v = 0$

For rubber $v = 0.5$

For concrete $v = 0.1 - 0.2$

Isotropic Material: These materials have same elastic properties in all directions.

No. of independent elastic constants = 2, *i.e.* if any of 2 elastic constants is known then other can be derived.

Orthotropic Material: The number of independent elastic constants is 9.

Anisotropic materials: These materials don't have same elastic properties in all directions.

Elastic modulii will vary with additional stresses appearing. ∵ There is a coupling between shear stress and normal stress for an isotropic material.

The number of independent elastic constants is 21.

Hooke's Law: It states that when a material is loaded such that the intensity of stress is within a certain limit, the ratio of the intensity of stress to the corresponding strain is a constant which is characteristics of that material.

i.e. $\boxed{\dfrac{\text{Stress}}{\text{Strain}} = \text{Constant} = E}$ *i.e.,* $\sigma = E\,\varepsilon$

Where, E = Young's Modulus (N/m^2)

Or

Modulus of Elasticity

- For steel, value of E = 210 GPa (1 GPa = 10^3 N/mm^2)
- For aluminum, value of E = 73 GPa $E_{Al} \simeq \frac{1}{3}$rd E_{steel}
- For Plastic, value of E = 1 GPa – 14 GPa

Note : As flexibility increases, value of young's modulus decreases.

It is resistance to elastic strain.

Shear Modulus of Elasticity OR Modulus of Rigidity (G or C): It is defined as the ratio of shearing stress to shearing strain.

$\boxed{G \text{ or } C = \dfrac{\text{Shear stress}}{\text{Shear strain}}}$ *i.e.* $\tau = G\phi$

Bulk Modulus (K):

It is defined as the ratio of uniform stress intensity to volumetric strain within the elastic limits.

$\boxed{K = \dfrac{\text{Stress}}{\text{Volumetric Strain}}}$

Note: Elastic constant relationship

(i) $E = 2C(1+v)$, where, v = Poisson's ratio.

(ii) $E = 3K(1-2v)$

(iii) $v = \dfrac{3K - 2C}{6K + 2C}$

(iv) $E = \dfrac{9KC}{3K + C}$

STRESS-STRAIN DIAGRAM:

1. Ductile material (Mild Steel):

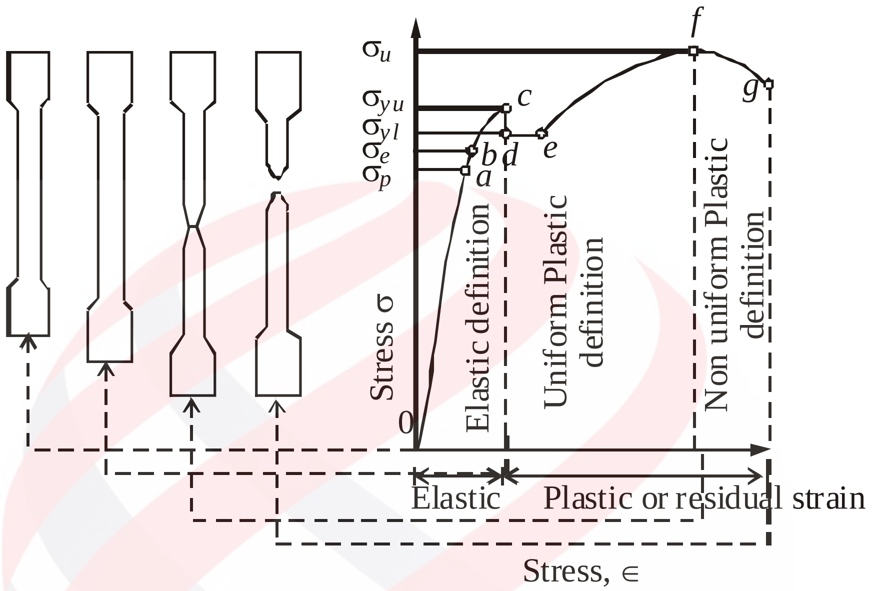


Figure: ***Typical stress-strain diagram for a ductile material***

- Point '*a*'→ Limit of proportionality: Up to this point '*a*', Hooke's law isobeyed; '*oa*' is a straight line. Stress corresponding to this point is called 'proportional limit stress, σ_p'
- Upto point a, Hooke's law is obeyed and stress is proportional to strain. Therefore, OA is straight line. Point A is called limit of proportionality.
- Point 'b'→ Elastic limit point: '*ab*' is not a straight line but upto point '*b*' the material remains elastic. Stress corresponding to this point is called elastic limit stress, σ_e.

Elastic limit > Proportional limit.

Generally, point '*a*' and '*b*' coincides.

- Point 'c'→ upper yield point: At this point the cross-sectional area starts decreasing.It initiates plastic deformation.
- Point 'c'→ Lower yield point. This is point of which, minimum stress is required to maintain plastic behavior.
- Point'd'→ Lower yield point: At this point the specimen elongates by a considerable amount without any increase in stress. The value of stress at this point is $\sigma_y = 250N/mm^2$ for mild steel.

The value of strain at yield stress is about 0.0012 or 0.12%

Lower yield point'd' is observed, if rate of loading is slow.

- Upper yield point 'c' is observed, if rate of loading is fast.
- Portion '*de*' represents 'plastic yielding': -Typical value of strain is 0.014 or 1.4% i.e. strain in range '*de*' is at least 10 times the strain at the yield point.
- Portion '*ef*' represents 'strain hardening': Strain increases fast with strain, till the ultimate load is reached.
- Point '*f*'→ Ultimate stress: Corresponding strain is 20% for mild steel. It is the maximum stress to which the material can be subjected in a simple tensile test. At this point necking of material begins.
- Point 'g' → Breaking Stress: - Corresponding strain is called fracture strain. It is about 25% for mild steal .

Concept of reduced area (RA): $q = \frac{A_f - A_o}{A_o}$

- Reduction of area is more a measure of deformation required to produce failure and its chief contribution results from necking process.
- There is a complicate state of stress in necking condition.

RA is the most sensitive ductility parameter and is useful in detecting quality changes in materials.

Comparison of Engineering and true stress strain curve:

- The true stress-strain curve is also known as **flow curve.**
- True stress-strain curve gives a true indication of deformation characteristics because it is **based on the instantaneous dimension of specimen.**
- In engineering stress-strain curve, the stress drops down after necking since it is **based on the original area.**
- In true stress strain curve, the stress however increases after necking since the cross section area of the specimen **decreases rapidly after necking.**
- The flow curve of many metals in the region of uniform plastic deformation can be expressed by **simple power law.**

$$\sigma_T = K(\in_T)^n$$

where, K is the strength co-efficient, σ_T is time stress.

n is the strain hardening coefficient.

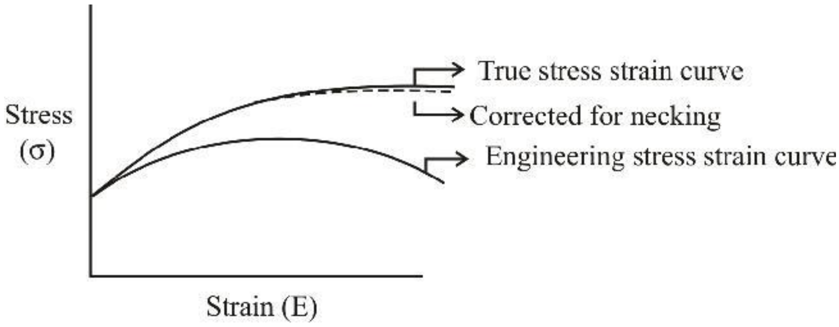
$n = 0$ for perfectly plastic solid

$n = 1$ In elastic solid

For most metals $0.1 < n < 0.5$

$\sigma_{True} > \sigma_{Nominal} \rightarrow$ if force is tensile, since area decreases.

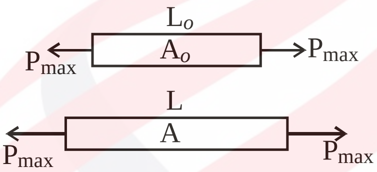
$\sigma_{True} > \sigma_{Nominal} \rightarrow$ if force is compressive, since area increase.



Relation between ultimate tensile strength and true stress at maximum load.

Ultimate tensile strength $\sigma_u = \frac{P_{max}}{A_o}$

True stress at maximum load = $(\sigma_u)_T = \frac{P_{max}}{A}$



True strain at max load $(\varepsilon_T) = \ln\frac{A_o}{A}$ or $\frac{A_o}{A} = e^{\epsilon_T}$

Eliminating P_{max} we get

$$(\sigma_u)_T = \frac{P_{max}}{A} \times \frac{A_o}{A_o}$$

$$= \frac{P_{max}}{A_o} \times e^{\epsilon_T}$$

$$\Rightarrow (\sigma_u)_T = \sigma_u\, e^{\epsilon_T}$$

Here, P_{max} is the max force.

A_o = original cross section area

A = instantaneous cross section area

- Based on the above theory two examples has been provided.

Example 1. Only elongation no neck formation.

In the tension test of rod shown initially it was $A_o = 50\,\text{mm}^2$ and $L_o = 100\,\text{mm}$. After the application of load its $A = 40\,\text{mm}^2$ and L = 125 mm.

Determine the true strain using changes in both length and area.

Solution: Here $A_o L_o = AL$

i.e., $50 \times 100 = 40 \times 125$

$\Rightarrow$ $5000\,\text{mm}^2 = 5000\,\text{mm}^2$ $\therefore$ no neck formation.

$\therefore$ true strain can be calculated both by area and length formula as follows.

$$\in_T = \int_{l_o}^{l} \frac{dl}{l} = \ln\left(\frac{125}{100}\right) = 0.223$$

$$\in_T = \int_{A_o}^{A} \ln\left(\frac{A_o}{A}\right) = \ln\left(\frac{50}{40}\right) = 0.223$$

Example 2: A ductile material is tested such that necking occurs then the final gauge length is L = 140 mm and the final minimum cross section area is $A = 35\,\text{mm}^2$ though the rod shown initially was of area $A_o = 50\,\text{mm}^2$ and $L_o = 100\,\text{mm}$. Determine the true strain using change in both length and area.

Sol. Check $A_o L_o = 50 \times 100 = 5000\,\text{mm}^3$

$AL = 35 \times 140 = 4900\,\text{mm}^3$

i.e. $A_o L_o > AL$ $\therefore$ Necking occurs and force applied is tensile.

$$\therefore \quad \in_T = \ln\left(\frac{A_o}{A}\right) = \ln\left(\frac{50}{35}\right) = 0.357$$

$$\in_T = \int_{l_o}^{l} \frac{dl}{l} = \ln\left(\frac{140}{100}\right) = 0.336 \text{ (wrong)}$$

Inference: After necking gauge length gives error but area and diameter can be used for the calculation of true strain at and before fracture.

It Elongation with neck formation.

2. Brittle Material (Cast Iron):

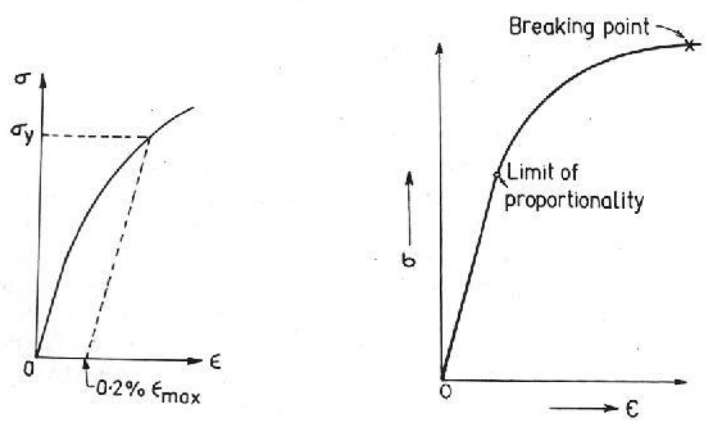


Figure: ***Typical stress-strain diagram for a ductile material***

- In these materials, elongation and reduction in area of the specimen is very small.
- The yield point is not marked at all.
- The straight portion of the diagram is very small.
- **Proof stress:** It is given corresponding to 0.2% of strain. A line parallel to linear portion of curve is drawn passing through 0.2% strain:

$$\sigma = \epsilon_{Total} E - \epsilon_{Plastic} E = \epsilon_{Elastic} \times E$$

Concept of Elastic and Plastic strain by graph: