BASIC CIRCUIT ELEMENTS AND THEORY

INTRODUCTION TO CIRCUIT ELEMENT:

Electric circuit is an inter connection of electric elements.

- Charge is electrical property of atomic particles.

  Unit $\rightarrow$ Coulombs
- Charge on electron = -1.6×10^{-19}C
- Current is time rate of change of charge.

$$I = \frac{dq}{dt} \quad \text{or} \quad q = \int_0^t I\,dt$$

$$\text{Unit} \rightarrow \frac{\text{C}}{\text{sec}} \text{ or Amp}$$

- Voltage is the energy required to move a charge from one point to another point.

$$V = \frac{dW}{dq}$$

$$\text{Unit} \rightarrow \frac{\text{J}}{\text{C}} \text{ or Volt}$$

- Power is the time rate of change of energy.

$$P = \frac{dW}{dt} \quad \text{or} \quad W = \int P\,dt$$

$$\text{Unit} \rightarrow \frac{\text{Joule}}{\text{sec}}$$

Classification of circuit element:-

(i) Unilateral and Bilateral element:-

If the element property and characteristic does not change with direction of current, then the element is called bilateral element; otherwise unilateral element.

(ii) Linear and non linear element:-

If the element satisfy homogeneity and additivity property then element is called linear element, otherwise non linear.

(iii) Active and passive Elements:

Active Elements: When the element is capable of delivering the energy, it is called active element.

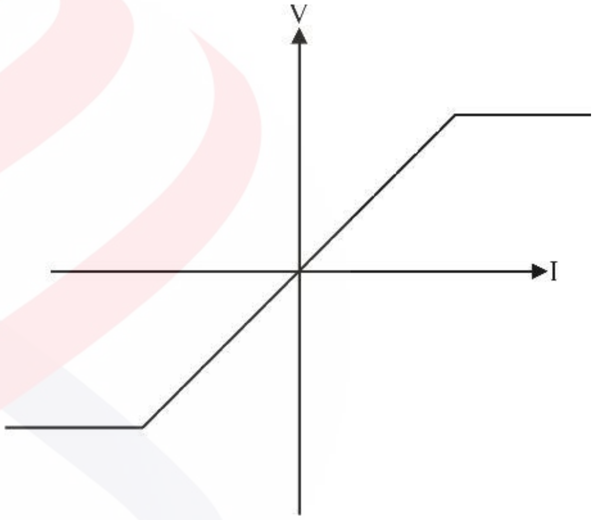
Example: Voltage source, Current source, Transistor, Diode, Op-amp etc

Passive Elements: When the element is not capable of delivering the energy, it is called passive element.

Example: Resistance, capacitor, inductor etc.

Example: Identify weather the element is:

i. Linear or non linear
ii. Active or passive
iii. Bilateral or unilateral



Solution:-

i. Nonlinear, as slope is not constant.
ii. Passive, as V/I is +ve in both quadrants.
iii. Bilateral, as characteristic is identical in opposite quadrant.

1. Resistance:

Ohm's law:- Voltage V across a resistor is directly proportional to the current i flowing through the resistor

$\Rightarrow$ $V \propto i$

$V = iR$, This constant of proportionality is called 'resistance'.

$\Rightarrow$ Case 1:- When R = 0 $\rightarrow$ short circuit

Then V = 0 and I = ∞

Case 2:- When R = ∞ $\rightarrow$ Open circuit

Then V = ∞ and I = 0

Key Points:

- Power in resistor is given by

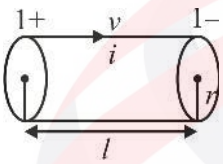
$$P = vi = i^2R = \frac{v^2}{R}$$

- Energy is then determined as the integral of instantaneous power as :

$$E = \int_{t_1}^{t_2} Pdt = R\int_{t_1}^{t_2} i^2dt = \frac{1}{R}\int_{t_1}^{t_2} v^2dt$$

- Resistor consumes energy and converts electrical energy into heat energy.
- Resistance depends on the geometry of material and also on nature of material as:

$$R = ° \frac{l}{A}$$



Where ° = Resistivity (Ω.m)
° = 1/σ (σ = conductivity)
Unit of conductivity: mho/m or Siemens/cm

- Resistivity of wire is materialistic property i.e. It does not vary with circuit geometry.
- Extension of wire result in increase in length & decrease in cross-sectional area therefore resistance of wire increases.
- When circuit is short circuit means, $R = 0$. When circuit is open, $R = \infty$.

Example: A conductor has a resistance of 3Ω. What is resistance of the same material? Which has one half the diameter and 4 times the length of the given conductor.

EXP: $R_1 = \rho\frac{l_1}{A_1}$, $R_2 = \rho\frac{l_2}{A_2}$

$$\Rightarrow \frac{R_2}{R_1} = \frac{l_2}{l_1}\frac{A_1}{A_2}$$

Now $A_1 = \frac{\pi D_1^2}{4}$, $A_2 = \frac{\pi D_2^2}{4}$

$$\Rightarrow \frac{A_1}{A_2} = \frac{D_1^2}{D_2^2} = \frac{D_1^2}{\left(\frac{D_1}{2}\right)^2} = 4 \text{ and}$$

$$l_2 = 4l_1 \Rightarrow \frac{l_2}{l_1} = 4$$

$$\Rightarrow \frac{R_2}{R_1} = 4 \times 4 = 16$$

$$\Rightarrow R_2 = 16 \times 3 = 48\Omega$$

2. CAPACITANCE:

Capacitance is the property of capacitor which opposes the sudden change in voltage.

$$Q \propto V$$

$$Q = CV$$

$$\frac{dQ}{dt} = C\frac{dv}{dt}$$

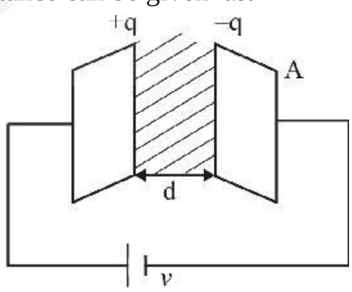
$$I = C\frac{dv}{dt}$$

$$\text{Or, } V = \frac{1}{C}\int_{-\infty}^{t} I\, dt$$

The circuit element that stores energy in an electric field is called capacitor.

Key Points:

(a) **<u>Capacitors retain the charge & thus electric field</u>** after removal of the source applied. (While inductors do not retain energy). For parallel plate capacitor, the capacitance can be given as:



$$C = \frac{\epsilon_o \epsilon_r A}{d}$$

Where A = cross-sectional area of plate
ϵ_r = Relative permittivity of dielectric
ϵ_o = Permittivity of free space

d = distance between plates

$$C = \frac{8.854 \in_r A}{d} pF$$

(b) The charge q on capacitor results in an electric field in the dielectric which is the mechanism of energy storage.

(c) Power and energy relation for capacitance are as:

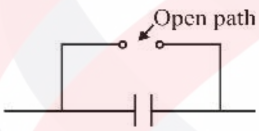
$$P = vi = vc\frac{dv}{dt} = \frac{d}{dt}\left[\frac{1}{2}cv^2\right] \qquad \left\{i = \frac{cdv}{dt}\right\}$$

$$P = \frac{d}{dt}\left[\frac{1}{2}cv^2\right]$$

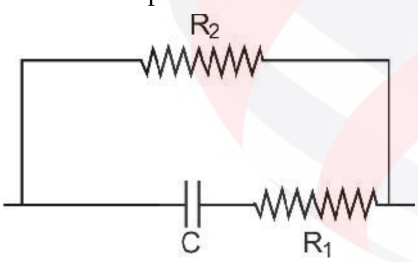
Energy $w_c = \int P\,dt = \int v\,c\frac{dv}{dt}dt$

(d) The energy stored in the electric field of capacitance is $w_c = \frac{1}{2}cv^2$

(e) Ideal capacitor:



Practical capacitor:



3. INDUCTANCE:

Inductor:- Inductance is the property of the inductor which opposes the sudden change in current.

Concept:- When a time varying current is flowing through the coil, then magnetic flux is induced and it is given by

$\cdot \propto I$

$\cdot = Li$

$N¤ = LI$

$$L = \frac{N¤}{I}$$

Key Points:

(a) The flux linkage across inductor is $N\phi$. Thus $N¤ = LI$

(b) **Proof of equation A:** According to faraday's law, the emf induced across a inductor is directly proportional to the rate of change of flux through it.

$e = -N\frac{d¤}{dt}$ {N = no of turns in the coil}

$$e = -N\frac{d}{dt}\left\{\frac{LI}{N}\right\}$$

$$e = -L\frac{dI}{dt}$$

-ve sign indicates the opposition caused by induced emf to change of flux (Lenz's Law)

(c) The power across the inductor is:

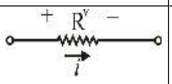
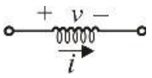
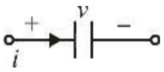
$$P = vi = L\frac{di}{dt}i = \frac{d}{dt}\left[\frac{1}{2}Li^2\right]$$

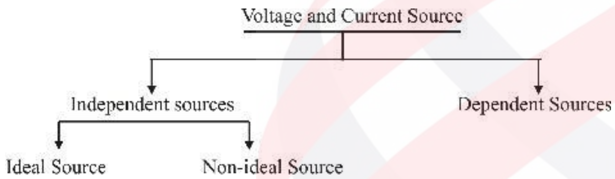
(d) Energy: $w = \int P\,dt = \int Li\,dt$

$$w = \frac{1}{2}L\left[i_2^2 - i_1^2\right]$$

Energy stored in magnetic field by inductor is $w = \frac{1}{2}Li^2$

Relationship of parameters:

Element	Units	Voltage	Current	Power
 Resistance	Ohms (Ω)	$v = Ri$ (ohms law)	$i = \frac{v}{R}$	$P = vi = i^2 R$
 inductance	Henry (H)	$v = L\frac{di}{dt}$	$i = \frac{1}{L}\int v\,dt$	$P = vi = i\frac{Ldi}{dt}$
 Capacitance	Farad (F)	$v = \frac{1}{c}\int idt$	$i = c\frac{dv}{dt}$	$P = vi = vc\frac{dv}{dt}$



Voltage & Current Source:

The sources are of two types, one is independent sources and other is dependent sources:

Independent sources:

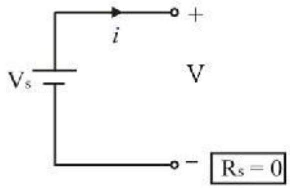
The voltage or current source in which the value of voltage or current remains constant, and does not vary with other circuit element.

Ideal & practical voltage sources:

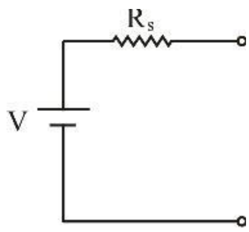
If the voltage source delivers energy at particular voltage, which is independent of source current then voltage source is ideal, otherwise practical.

Ideal:- $V = V_S$

Practical:- $V = V_S - IR_S$



Ideal Voltage Source



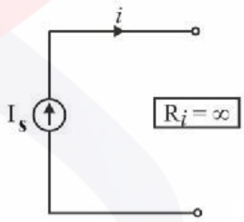
Practical Voltage source

Ideal & practical voltage current sources:

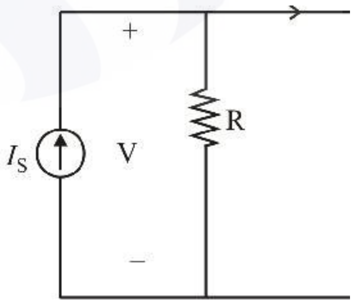
If the current source delivers energy at particular value, which is independent of source voltage, then current source is ideal, otherwise practical.

Ideal: $I = I_S$

Practical: $I = I_S - V/R_S$



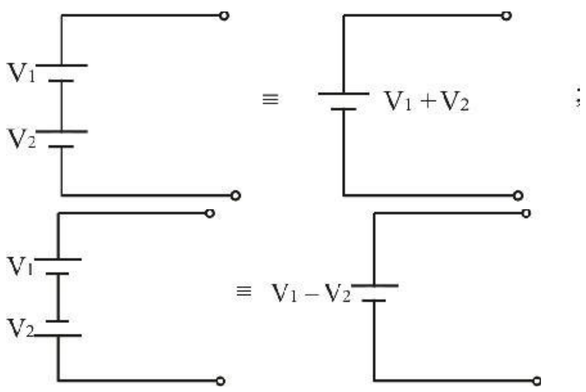
Ideal current source



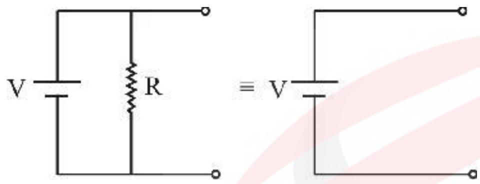
Practical Current source

(a) In non ideal voltage source, the internal resistance of voltage source is of finite value and is always in series with voltage source.

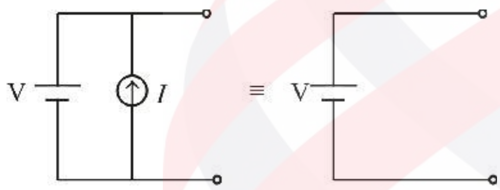
(b) In non ideal current source, the internal resistance of current source is of finite value & is always in parallel with current source.



(c)



(d)



Key Points:

→ Resistance in parallel with a voltage source is redundant as the terminal voltage remains same.

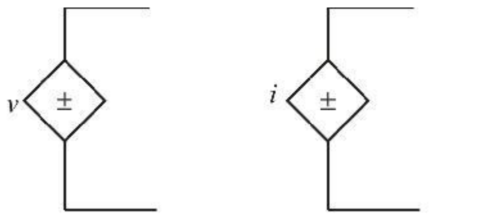
→ Resistance in series with the current source is redundant as the short circuit current in loop is independent of value of R.

→ When current sources are connected in series they should all have same value.

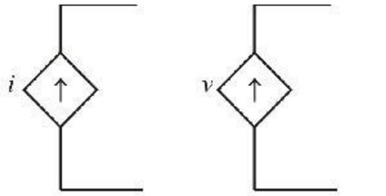
→ When voltage sources are connected in parallel they should have same value.

Dependent Voltage and Current Sources:

These are voltage and current sources whose value do not remain constant, rather varies with circuit elements or independent sources:



Voltage dependent voltage source   Current dependent voltage source



Current dependent current source   Voltage dependent current source

Distributed and Lumped Network:

In Lumped network, we can separate resistance, inductance, and capacitance separately or single element in one location is used to represent a distributed resistance.

Example: A coil having large number of turns of insulated wire has resistance throughout the length of wire but only resistance at single plane represents the distributed resistance.

In Distributed network, the circuit elements are not at one location rather they are distributed.

Example: Transmission line, the resistance, inductance and capacitance are distributed throughout the length of Transmission line.

Note: In distributed network, the circuit elements are represented as per unit length.

Non Linearity of circuit elements:

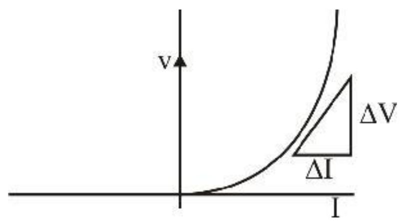
1. **Resistance Non Linearity:** If the current voltage relationship in an element is not linear, then the element is modeled as non linear resistor.

   Example: Diode, filament lamp

   (a) The non linear resistance can be given as:

   $$R = \frac{\Delta V}{\Delta I}$$

   Also called as A.C. resistance



Note: Ohm's law is valid for linear circuit elements. Also it is not valid for open circuit element because for open circuit:

$I = 0, R = \infty$

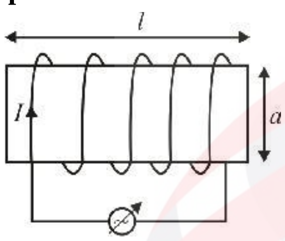
So $V = \infty$

$$\boxed{V \neq IR}$$

2. **Inductors non linearity:** When the inductance of inductor depends on the current magnitude, then the inductor is called non linear inductor:

Example: Iron core inductor.

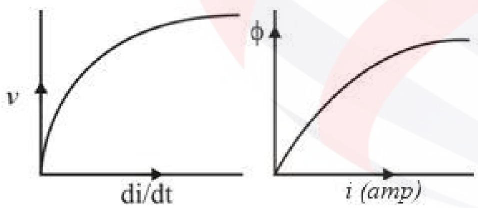
Key points:



(a) $\boxed{N¤ = LI \Rightarrow L = \frac{N¤}{I} = Variable}$

(b) Also we know;

$$\boxed{V = L\frac{di}{dt} \Rightarrow L = \frac{V}{di/dt} = Variable}$$



(1) (2)

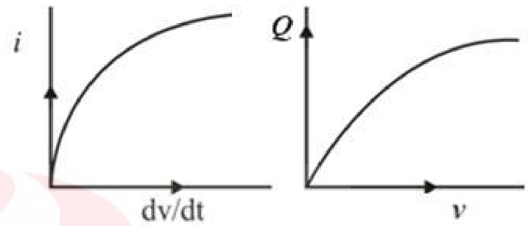
As the slope of the curve in both cases is **L** (inductance) and **L** is variable. So, the curve is not linear.

The second curve shows that after certain value of current, the flux does not increase due to saturation of iron core.

3. **Nonlinearity in capacitance:** When the capacitance of capacitor depends on voltage magnitude, then capacitor is called non linear capacitor.

$$\boxed{Q = CV \Rightarrow C = \frac{Q}{V} = Variable}$$

$$\boxed{i = C\frac{dv}{dt} \Rightarrow C = \frac{i}{dv/dt} = Variable}$$



(1) (2)

As the slope of the curve in both cases is **C** and **C** is variable. So, the curve is not linear.

Key Points:

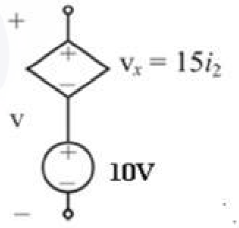
(*a*) Resistances may exhibit non linearity and not obey ohm's law due to thermal effects.

(*b*) Inductances without air core have saturation characteristics hence they lose their linearity.

Example: Obtain the voltage V in the branch shown in figure for

(a) $i_2 = 1A$ (b) $i_2 = -2A$ (c) $i_2 = 0A$.

Solution: $v = 10 + v_x$ for



(a) $i_2 = 1A$

$V = 10 + 15 = 25V$

(b) $V = 10 - 15 \times 2 = -20V$.

(c) $V = 10 + 15 \times 0 = 10V$.

SSC-JE PRACTICE SET

1. Ohm's law in point form in field theory can be expressed as

(*a*) V = RI (*b*) $J = \frac{E}{\sigma}$ (*c*) $J = \sigma E$ (*d*) $R = \rho\frac{L}{A}$

Ans. **(*c*)**

2. Which of the following is a non linear device.

(*a*) Resistor (*b*) Diode (*c*) Inductor (*d*) Capacitor

Ans. **(*b*)**

3. Which of the following is not a bilateral element

(*a*) Current source (*b*) Resistor
(*c*) Inductor (*d*) Capacitor

Ans. **(*a*)**

4. **Consider two metallic wires W_1 and W_2. They are made up of same material and each has circular cross section. The diameter of W_2 is twice that of W_1 and the length of W_2 is four times that of W_1. Which one of the following statements is TRUE?**

(a) Resistance of W_1 is half that of W_2

(b) Resistance of W_1 is equal to that of W_2

(c) Resistance of W_1 is twice that of W_2

(d) Resistance of W_1 is eight times that of W_2

ANS: (b)

EXP: Given: $D_2 = 2D_1$ and $l_2 = 4l_1$

We know that,

Resistance of wire is given by,

$$R = \frac{\rho l}{A} \qquad \{\rho \rightarrow \text{resistivity}\}$$

$$R = \frac{\rho l}{\pi D^2/4}$$

$$\Rightarrow R_1 = \frac{\rho l_1}{\pi D_1^2/4}$$

$$\Rightarrow R_2 = \frac{\rho l_2}{\pi D_2^2/4} = \frac{\rho 4l_1}{\pi 4D_1^2/4}$$

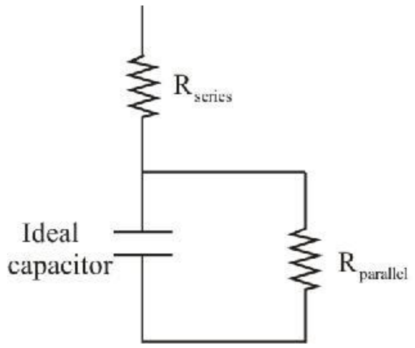
$$\Rightarrow R_2 = \frac{4l_1\rho}{\pi D_1^2} = R_1$$

5. The series equivalent resistance value in case of a lossy capacitor will be

(a) Very small (b) Small (c) Large (d) None of these

ANS: (b)

EXP: Equivalent circuit for a lossy capacitor:



Here, R_{Series} is low and $R_{Parallel}$ is high

6. A capacitor of capacitance C_1 and distance between the plate is d_1. A second capacitor of capacitance C_2 and distance between the plate is d_2. When they are connected to series what is the equivalent capacitance?

(a) $\frac{d_1 d_2}{d_1 + d_2}$ (b) $\frac{d_1 + d_2}{d_1 d_2}$ (c) $C_1 + C_2$ (d) $\frac{C_1 C_2}{C_1 + C_2}$

Ans. (d)

EXP: We know that, When two capacitors are is series S. The equivalent capacitance is

$$C_{eq} = \frac{C_1 C_2}{C_1 + C_2}$$

7. An ideal voltage source should have:
(a.) Infinite source resistance
(b.) Large value of emf
(c.) Small value of emf
(d.) Zero source resistance

ANS: d

8. To neglect a current source, the terminals across the source are
(a.) Open-circuited
(b.) Short-circuited
(c.) Replaced by some resistance
(d.) Replaced by capacitance
ANS: a

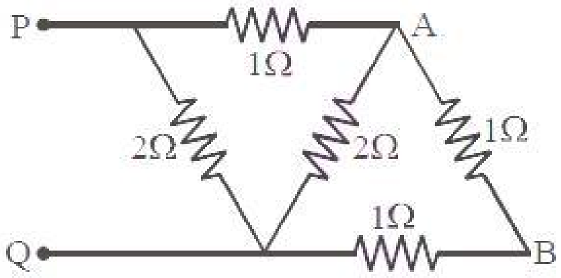
9. If two capacitances C_1 and C_2 are connected in parallel then the equivalent capacitance is given by

(SSC JE-2015)

(a) $C_1 C_2$ (b) $\frac{C_1}{C_2}$ (c) $\frac{C_1 C_2}{C_1 + C_2}$ (d) $C_1 + C_2$

Ans: d

10.



For the circuit shown find the resistance between points P & Q **(SSCJE-2015)**

(a) 1 Ω (b) 2 Ω (c) 3 Ω (d) 4 Ω

Ans: a

11. The rate of change of current in a 4 H inductor is 2 Amps/ sec. Find the voltage across inductor.
(a) 8 V (b) 0.8 V (c) 2 V (d) 16 V

Ans.(a)

12. How much energy is stored by a 100 mH inductance when a current of 1 A is flowing through it ?
(a) 0.5 J (b) 0.05 J (c) 0.005 J (d) 5.0 J **(SSC-JE-2015)**

Ans.(b)

13. What is the Power consumed by the resistor of 20 Ω connected across 100 V source ?
(a) 500 W (b) 50 W (c) 100 W (d) 300 W

Ans.(a)

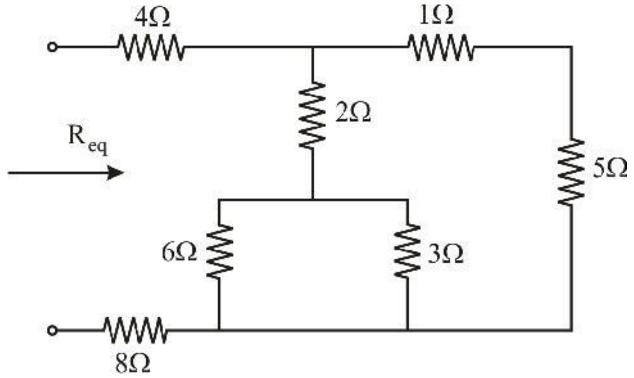
14. A linear circuit is one whose parameters **(SSC-JE-2015)**
(a) change with change in current
(b) change with change in voltage
(c) do not change with voltage and current
(d) None of the above

Ans. (c)

15 An active element in a circuit is one which **(SSC-JE-2015)**
(a) supplies energy
(b) receives energy
(c) dissipates energy
(d) both receives and supplies energy

Ans. (a)

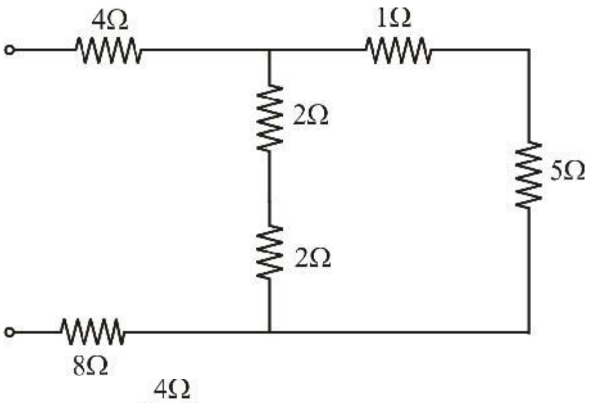
16. The R_{eq} for the circuit shown in figure is: **(SSC-JE-2014)**

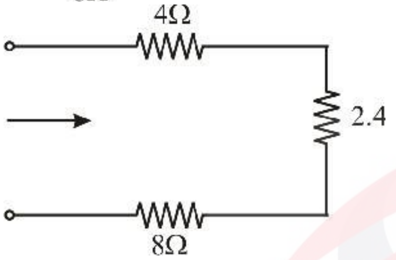


(a) 14.4Ω (b) 14.57Ω (c) 15.27Ω (d) 15.88Ω

ANS: a

EXP:





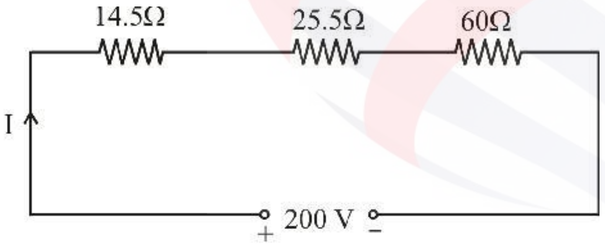
$R_{eq} = 4+8+2.4 = 14.4$

17. The SI unit of conductivity is: **(SSCJE-2014)**

(a.) ohm-m (b) ohm/m (c) mho-m (d) mho/m

ANS: d

18. Calculate the voltage drop across 14.5Ωresistance **(SSCJE-2014)**



(a.) 14.5V (b) 18V (c) 29V (d) 30.5V

ANS: c

EXP: $V = \frac{14.5}{14.5+25.5+60} \times 1200 = 29\,V$