

ELECTROSTATICS

Electrostatics fields: An electrostatic field is produced by a static charge distribution.

There are two fundamental laws governing Electrostatic fields

1. Coulomb's Law
2. Gauss's Law

1. Coulomb's Law:

Coulomb's Law states that the force F between two point charges Q_1 and Q_2 is

- i. Along the line joining them.
- ii. Directly proportional to the product $Q_1 Q_2$ of the charges.
- iii. Inversely proportional to the square of the distance R between them.

$$F_{12} = \frac{K Q_1 Q_2}{R^2} a_{R_{12}}$$

F_{12} → Force on Q_2 due to Q_1

F_{21} → Force on Q_1 due to Q_2

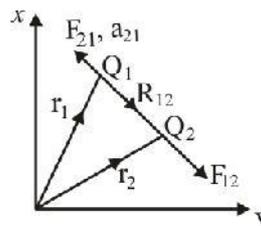
$$R_{12} = r_2 - r_1$$

$$|R_{12}| = R$$

$$a_{R_{12}} = \frac{R_{12}}{|R_{12}|} = \frac{R_{12}}{R}$$

$$F_{12} = \frac{K Q_1 Q_2}{R^2} \times \frac{R_{12}}{R} = \frac{K Q_1 Q_2}{R^3} R_{12}$$

$$F_{12} = \frac{K Q_1 Q_2 (r_2 - r_1)}{|r_2 - r_1|^3}$$



Where, $K = \text{proportionality constant (electrostatics constant)} = \frac{1}{4\pi\epsilon_0} = 9 \times 10^9 \text{ m/F}$

$\epsilon_0 = \text{permittivity of free space (F/m)}$

$$= 8.854 \times 10^{-12} \text{ F/m} = \frac{10^{-9}}{36\pi} \text{ F/m}$$

Electric field intensity: The electric field intensity is defined as the force per unit positive charge that would be experienced by a stationary point charge, or "test charge", at a given location in the field

- Its direction will be same as that of Electric force.

$$\boxed{E = \frac{F}{Q}} \quad \text{N/Coulomb or Volts/Meter}$$

F is the electric force experienced by the test particle.

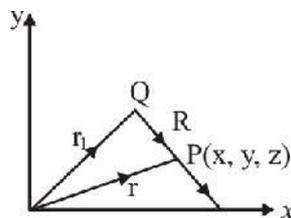
Q is the charge of the test particle in the electric field.

E is the electric field wherein the particle is located.

Electric field intensity at point r due to point charge located at r^1 is:

$$E = \frac{Q}{4\pi\epsilon_0 R^2} a_R$$

$$= \frac{Q (r - r^1)}{4\pi\epsilon_0 |r - r^1|^3}$$



For N point charges, $Q_1, Q_2, Q_3, \dots, Q_N$ located at r_1, r_2, \dots, r_N the Electric field intensity at point r is given by

$$E = \frac{Q_1(r-r_1)}{4fV_0|r-r_1|^3} + \frac{Q_2(r-r_2)}{4fV_0|r-r_2|^3} + \dots + \frac{Q_N(r-r_N)}{4fV_0|r-r_N|^3} = \frac{1}{4fV_0} \sum_{k=1}^N \frac{Q_k(r-r_k)}{|r-r_k|^3}$$

Electric flux and electric flux density:

Electric flux density or Electric displacement.

The Electric Flux Density is proportional to the Electric Field.

The proportionality constant depends on the medium being analyzed, and is known as the permittivity:

$$\vec{D} = \epsilon_0 \vec{E} \quad C/m^2$$

- The permittivity is often frequency-dependent, and is sometimes anisotropic (implying the permittivity depends on which direction the fields are in)

Electric flux: \mathcal{E}

The electric flux through an area is defined as the electric field multiplied by the area of the surface projected in a plane perpendicular to the field.

The electric flux through a planar area is defined as the electric field times the component of the area perpendicular to the field. If the area is not planar, then the evaluation of the flux generally requires an area integral since the angle will be continually changing.

$$\mathcal{E} = \int_S \vec{D} \cdot \vec{ds} \quad \text{in } C$$

One line of electric flux emanates from +1C and terminates on -1C, so it is measured in coulombs.

Gauss's Law – Maxwell's Equation:

Gauss's Law States that the total of the electric flux out of a closed surface is equal to the charge enclosed divided by the permittivity. $\oint \mathbf{E} \cdot d\mathbf{s} = \frac{Q_{enclosed}}{\epsilon_0}$

$$\oint_S \mathbf{D} \cdot d\mathbf{s} = Q_{enclosed} = \int_V \rho_v dV \quad \dots(i)$$

From divergence theorem

$$\oint_S \mathbf{D} \cdot d\mathbf{s} = \int_V \nabla \cdot \mathbf{D} dV \quad \dots(ii)$$

From equation (i) and (ii)

$$\rho_v = \nabla \cdot \mathbf{D}$$

First Maxwell's Equation i.e. volume charge density is the same as the divergence of Electric flux density.

Gauss Law is applicable to both symmetrical charge distribution, and also when charge distribution is not symmetrical.

But E or \vec{D} is determine from Gauss Law only for symmetrical charge distribution.

ELECTRIC FIELDS IN MATERIAL SPACE

As Electric fields exist in free space, they can exist in material media also.

Materials are classified in terms of their electrical properties (*conductivity*) as

- a. Conductor or metal
- b. Nonconductor or insulators or dielectric
 1. No conducting materials are usually referred to as insulator or dielectric.
 2. The conductivity of a material usually depends on temperature and frequency.
 3. A material with high conductivity ($\sigma \gg 1$) is referred to as a metal, where as one with low conductivity ($\sigma \ll 1$) is referred as insulator.

4. A material whose conductivity lies somewhere between those of metal and insulator is called a semiconductor.
5. The conductivity of metal generally increases with decrease in temperature. At temperatures near absolute zero ($T=0K$), some conductor exhibit infinite conductivity and are called superconductor.

Insulators	Semiconductor	Metal	Superconductor
rubber and glass	silicon and germanium	copper and aluminum	lead and aluminum

Convection and conduction currents

Currents: The current through a given area is the electric charge passing through the area per unit time.

$$I = \frac{dW}{dt} \text{ ampere (amp)}$$

Thus, in a current of one ampere, charge is being transferred at a rate of one coulomb per second.

The total current through a surface S is

$$I = \int_s J \cdot ds$$

Where J = current density A/m^2

J : The current density at a given point is the current through a unit normal area at that point.

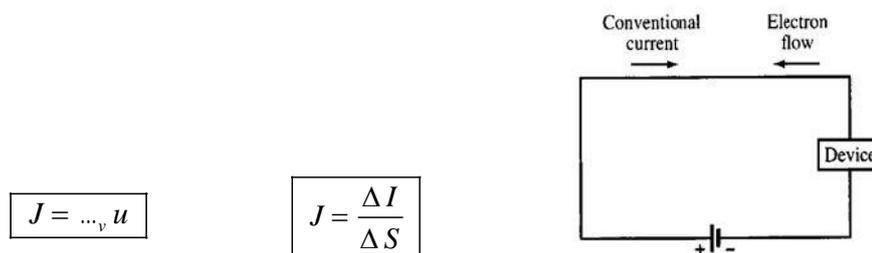
Depending on how I is produced, there are different kinds of current density.

- a. Convection current density.
- b. Conduction current density.
- c. Displacement current density.

Convection current:

Convection current does not involve conductors and so does not satisfy ohm's Law. It occurs when current flow through an insulating medium such as liquid, rarefied gas, vacuum,

The direction of CONVENTIONAL CURRENT is in the direction in which positive charge flows. In gases and liquids both positive and negative ions move. Only negative charges, i.e., electrons, move through solids and this are referred to as ELECTRON CURRENT.



$$J = \dots, u$$

$$J = \frac{\Delta I}{\Delta S}$$

J - Convection current density. I - conduction current density.

u - velocity of charge

Conduction current:

Conduction current requires a conductor. A conduction is characterized by a large no of free e's that provide conduction current due to impressed Electric field.

$$J = \uparrow E \text{ Point form of ohm's law.}$$

Conductor:

A conductor has an abundance of charge that is free to move. Consider an isolated conductor, when an external electric field E_e is applied to this conduct the +ve free charges are pushed along the same direction as the applied field while the negative free charges move in the opposite direction, and this charge migration takes place very quickly so the free charges accumulate on the surface of the conductor and from an *induced surface charge*; and this induced charges set up an internal induced field E_i this induced field cancels the external applied field E_e . So

A perfect conductor $\uparrow = \infty$ cannot contain an electrostatics field within it.

